

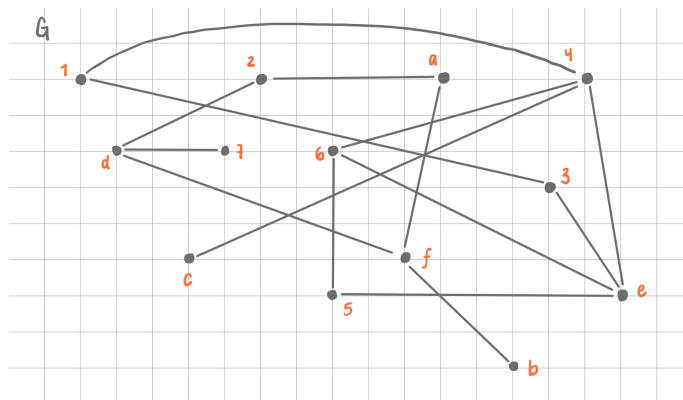


# Grade 7/8 Math Circles

February 19th, 2024

## Graph Theory: Isomorphisms - Problem Set

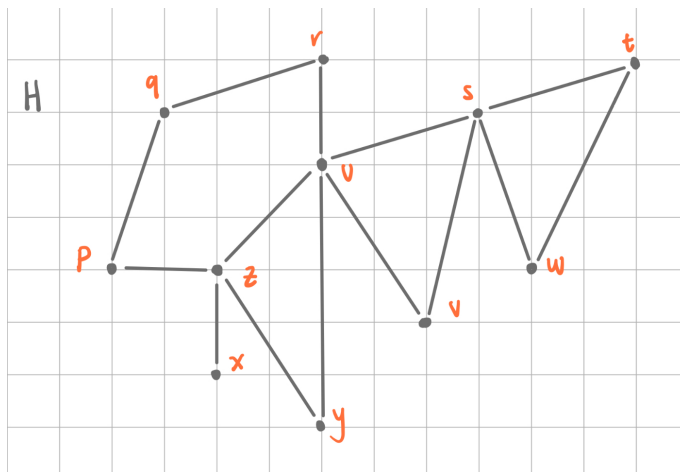
For the first four questions consider the graph  $G$  below:



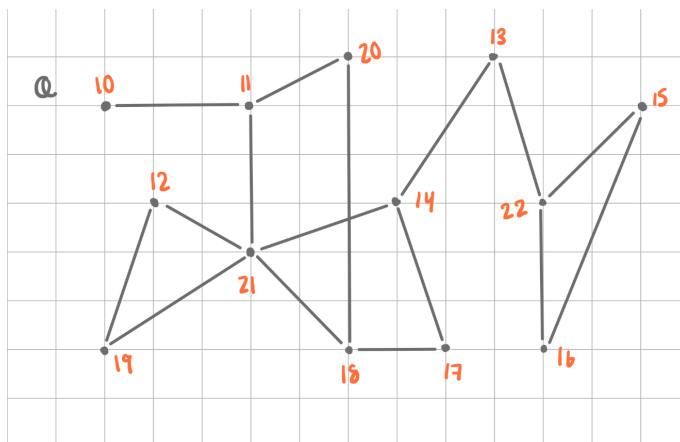
1. For the graph  $G$  answer the following questions:

- (a) What is  $V(G)$ ?
- (b) What is  $E(G)$ ?
- (c) What are the neighbours and degree of each vertex in  $G$ ?
- (d) How many components does  $G$  have?

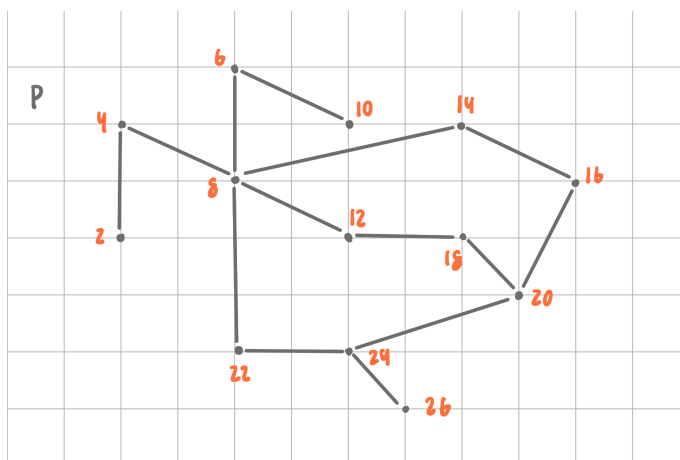
2. Is  $G$  isomorphic to the graph  $H$  below? If yes provide an isomorphism if not explain why.



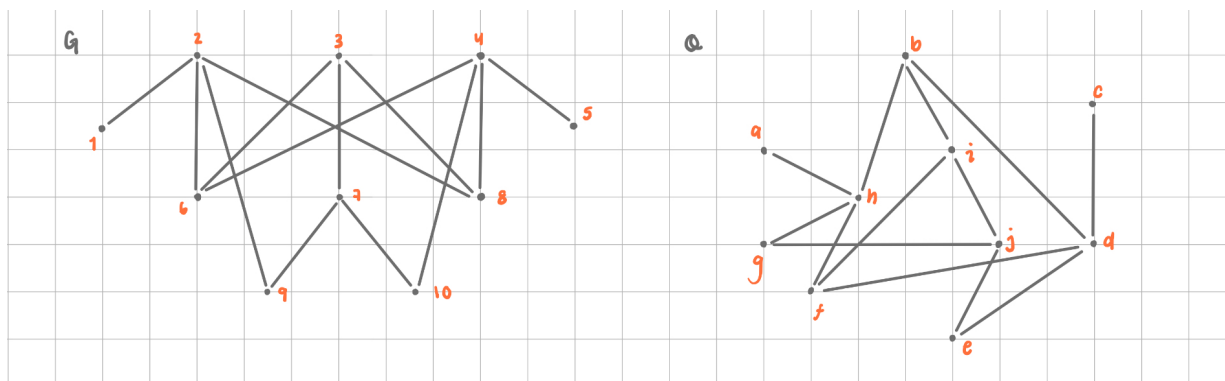
3. Is  $G$  isomorphic to the graph  $Q$  below? If yes provide an isomorphism if not explain why.



4. Is  $G$  isomorphic to the graph  $P$  below? If yes provide an isomorphism if not explain why.



For the next 4 Questions consider the **isomorphic** graphs  $G$  and  $Q$  below :



5. Is  $f : V(G) \rightarrow V(Q)$  an isomorphism, where  $f$  is the following map? If it is an isomorphism then prove it, if not then explain why:

$v$	1	2	3	4	5	6	7	8	9	10
$f(v)$	$a$	$b$	$c$	$d$	$e$	$f$	$g$	$h$	$i$	$j$



6. Is  $f : V(G) \rightarrow V(Q)$  an isomorphism, where  $f$  is the following map? If it is an isomorphism

$v$	1	2	3	4	5	6	7	8	9	10
$f(v)$	$a$	$b$	$c$	$d$	$e$	$a$	$g$	$h$	$i$	$j$

then prove it, if not then explain why:

7. Is  $f : V(G) \rightarrow V(Q)$  an isomorphism, where  $f$  is the following map? If it is an isomorphism

$v$	1	2	3	4	5	6	7	8	9	10
$f(v)$	$a$	$h$	$i$	$d$	$c$	$b$	$j$	$f$	$g$	$e$

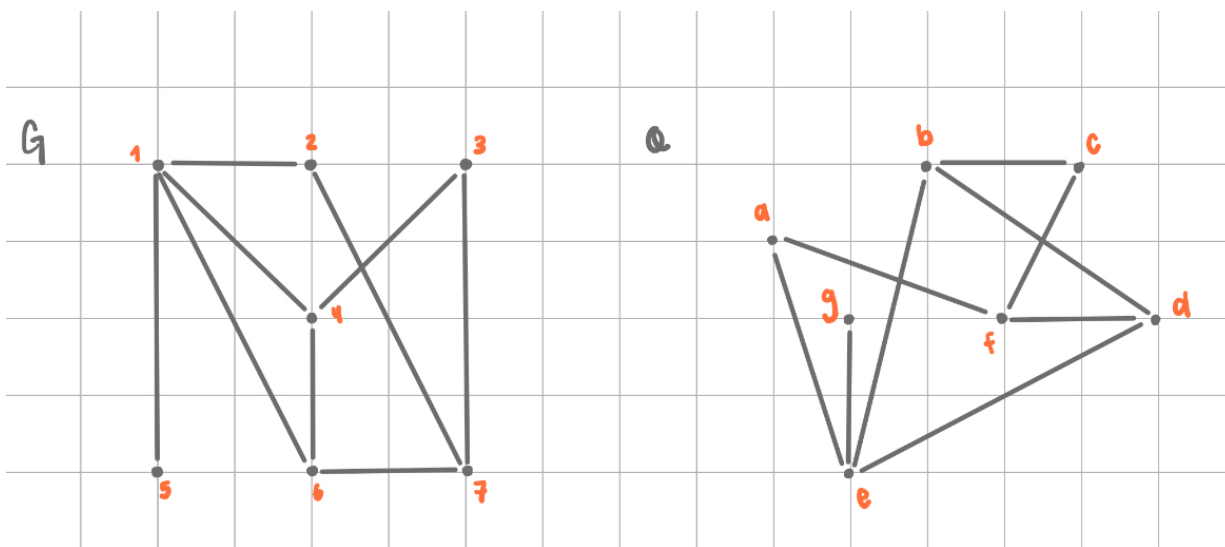
then prove it, if not then explain why:

8. Is  $f : V(G) \rightarrow V(Q)$  an isomorphism, where  $f$  is the following map? If it is an isomorphism

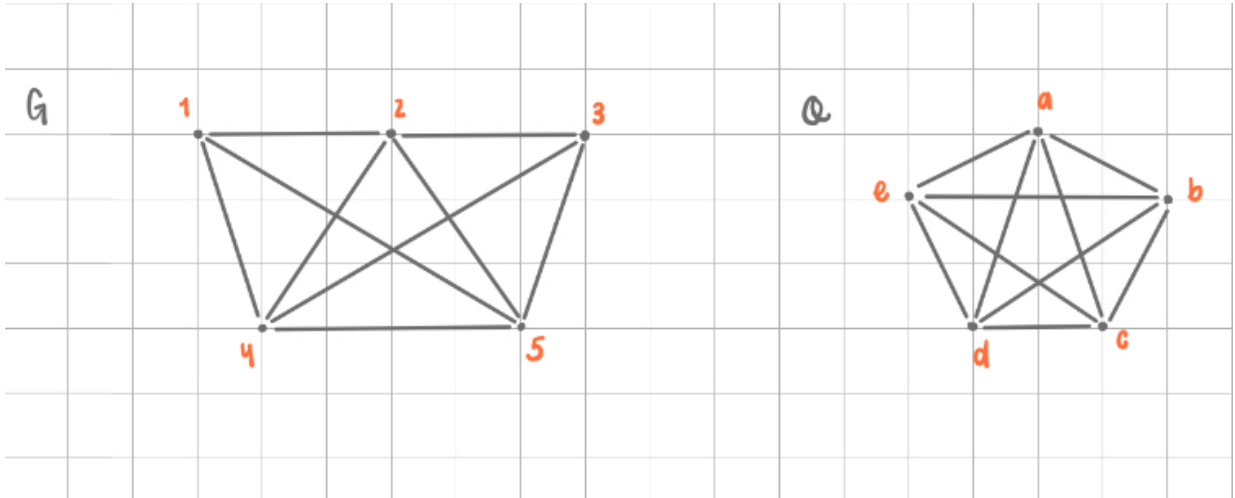
$v$	1	2	3	4	5	6	7	8	9	10
$f(v)$	$b$	$a$	$f$	$i$	$c$	$j$	$g$	$h$	$d$	$e$

then prove it, if not then explain why:

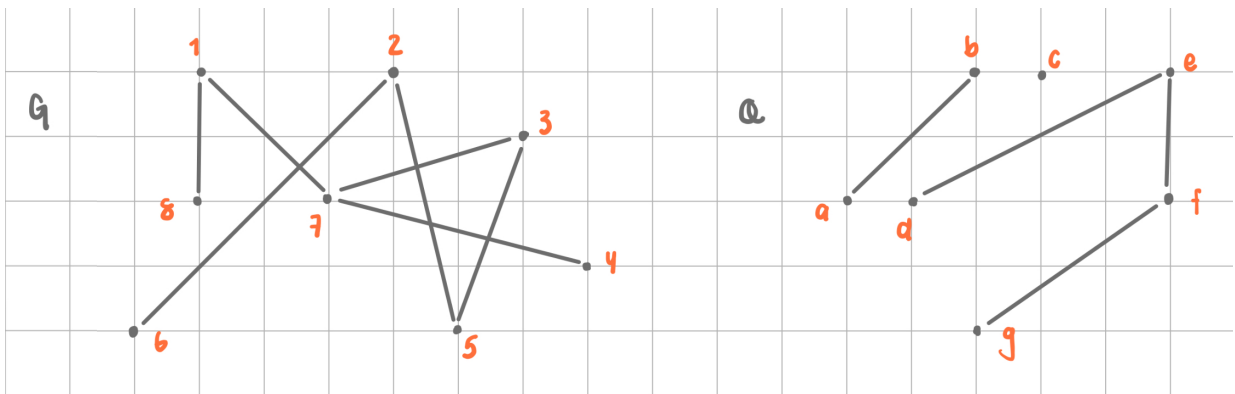
9. Are the following two graphs  $G$  and  $Q$  isomorphic? If yes provide an isomorphism and its inverse, if not then state why.



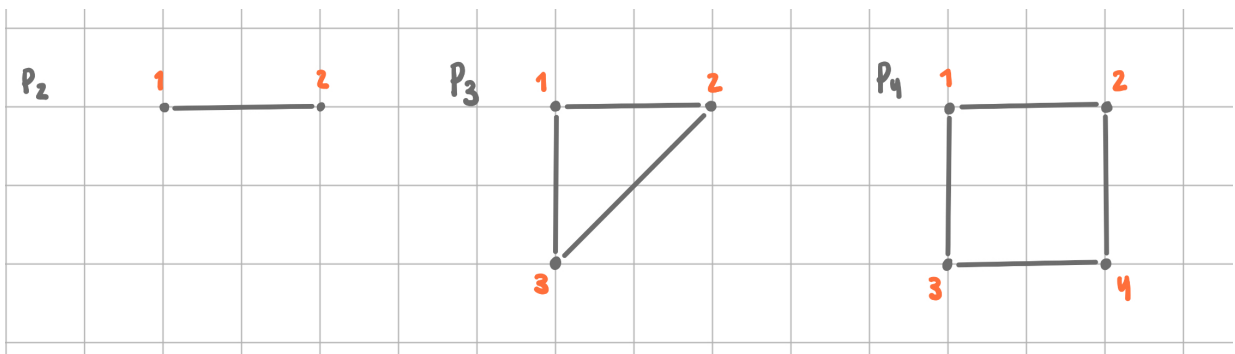
10. \* The following two graphs  $G$  and  $Q$  not isomorphic. With one change how could you make these two graphs isomorphic? Prove that after the change the graphs are isomorphic.



11. \* The following two graphs  $G$  and  $Q$  not isomorphic. With one change how could you make these two graphs isomorphic? Prove that after the change the graphs are isomorphic.

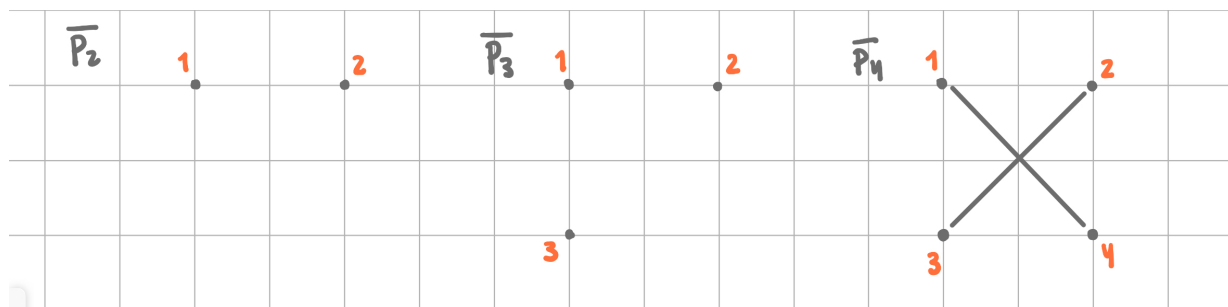


12. \*\*\* Below are the graphs  $P_2$ ,  $P_3$ , and  $P_4$  from the family of Polygon Graphs, the polygon graph  $P_n$  is simply the regular polygon with  $n$  sides ( $P_3$  is a triangle,  $P_4$  is a rectangle,  $P_5$  is a pentagon etc):





- (a) Draw and label the graphs  $P_5$ ,  $P_6$ , and  $P_7$ .
- (b) We define the complement of a graph  $G$  as  $\bar{G}$  to be a graph with the same vertex set as  $G$ , but has an edge set in which any edge that is not in  $G$  is an edge of  $\bar{G}$ . Below are the graphs of  $\bar{P}_2$ ,  $\bar{P}_3$ , and  $\bar{P}_4$ . Draw and label the graphs of  $\bar{P}_5$ ,  $\bar{P}_6$ , and  $\bar{P}_7$ .



- (c) Which of  $P_2$ ,  $P_3$ ,  $P_4$ ,  $P_5$ ,  $P_6$ , and  $P_7$  are isomorphic to their complement, state which one(s) are isomorphic and provide an isomorphism.
- (d) Besides the isomorphic graph(s) you found in part c is there any other graph in the Polygon Graph family which will be isomorphic to its complement? Explain your reasoning.